國立高雄科技大學
National Kaohsiung University of Science and Technology

Histogram Matching (Specification)

Speaker: Shih-Shinh Huang

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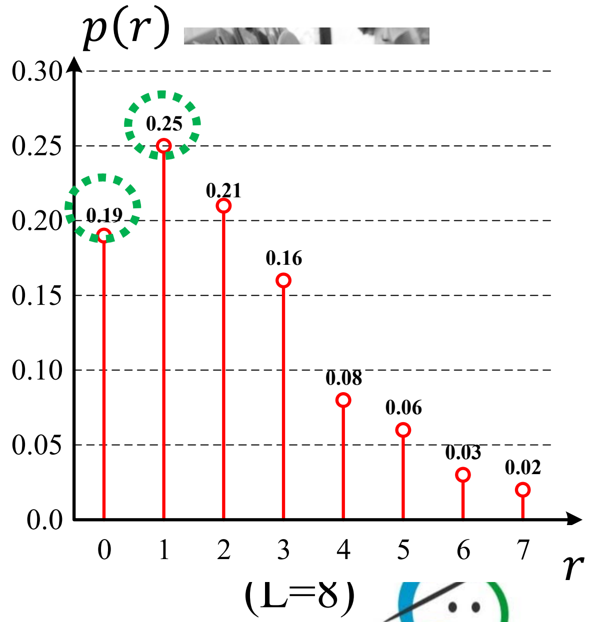
Outline

- Introduction
- Histogram Equalization
- Matching Algorithm

Introduction

- About Histogram
  - Definition: a discrete function $p(r)$ with $r = 0,1,\ldots L-1$ for L-level digital image.

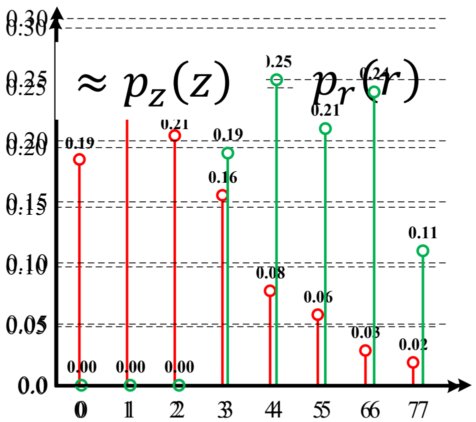
$$p(r = k) = \frac{n_k}{n}$$

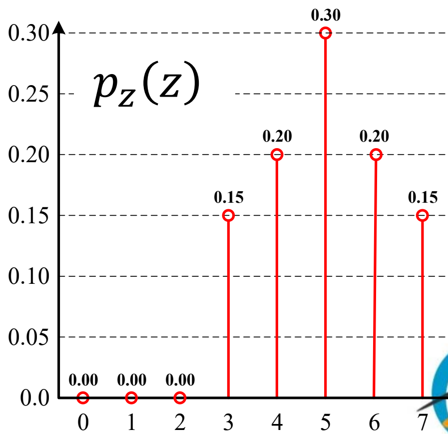


- n_k: number of points with gray level $r = k$
- n: total number of points

Introduction

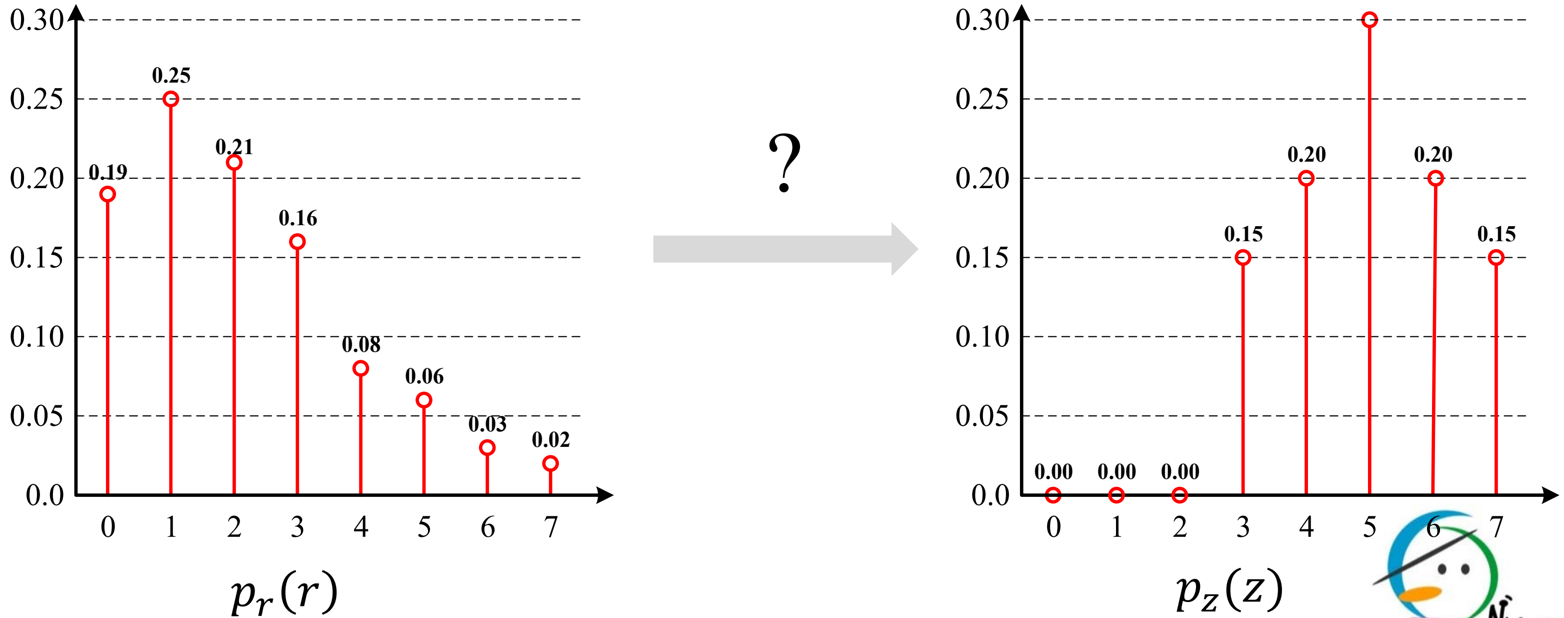
- About Histogram Matching
  - give an image with histogram $p_r(r)$ and a target histogram $p_z(z)$
  - generate a processed image that has a histogram approaching to $p_z(z)$





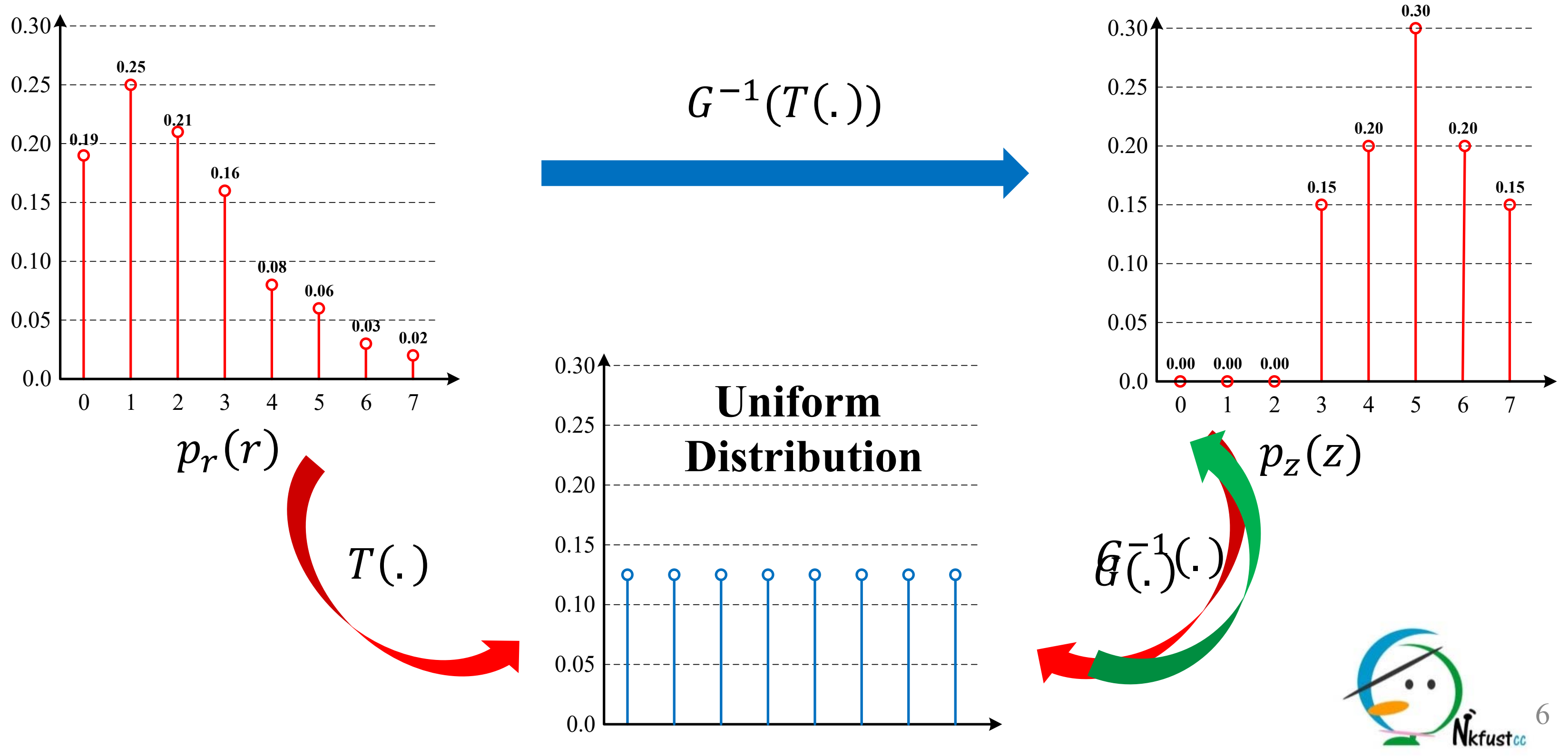
Introduction

- About Histogram Matching
  - The problem is how to find a transformation from $p_r(r)$ to $p_z(z)$



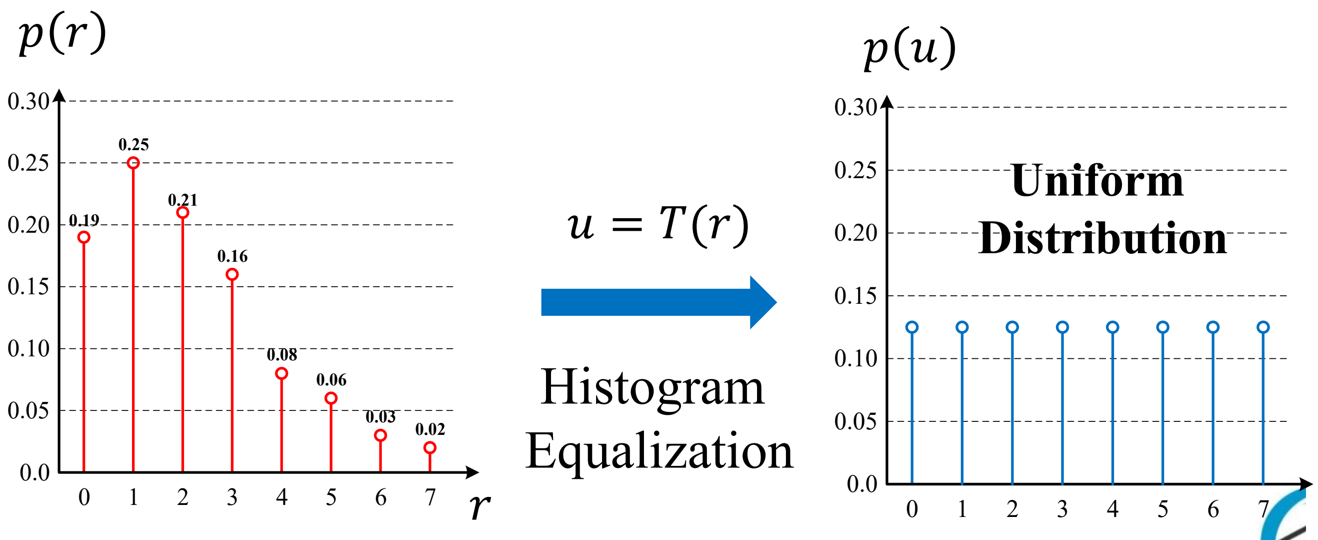
Introduction

- Idea of Histogram Matching
  - transform $p_r(r)$ to $p_z(z)$ via uniform distribution.



Histogram Equalization

- Description
  - give a histogram distribution $p(r)$
  - design a transformation $u = T(r)$ to equalize $p(r)$



Histogram Equalization

- Transformation Design

cumulative distribution function

$$T(r=k) = \boxed{(L-1)} \times \boxed{\sum_{j=0}^{k} p(r=j)}$$

maximal intensity

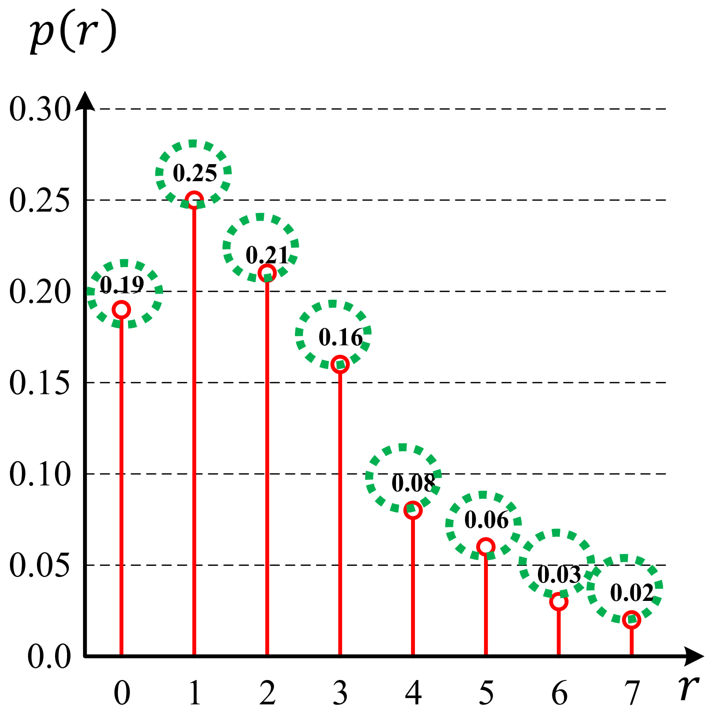
$$T(r=0) = (L-1) \times p(r=0)$$

$$T(r=1) = (L-1) \times \{p(r=0) + p(r=1)\}$$

$$\vdots \qquad \vdots$$

Histogram Equalization

- Example: $T(.): p(r) \rightarrow$ uniform



$$T(r=k) = (L-1) \times \sum_{j=0}^{k} p\,(r=j)$$

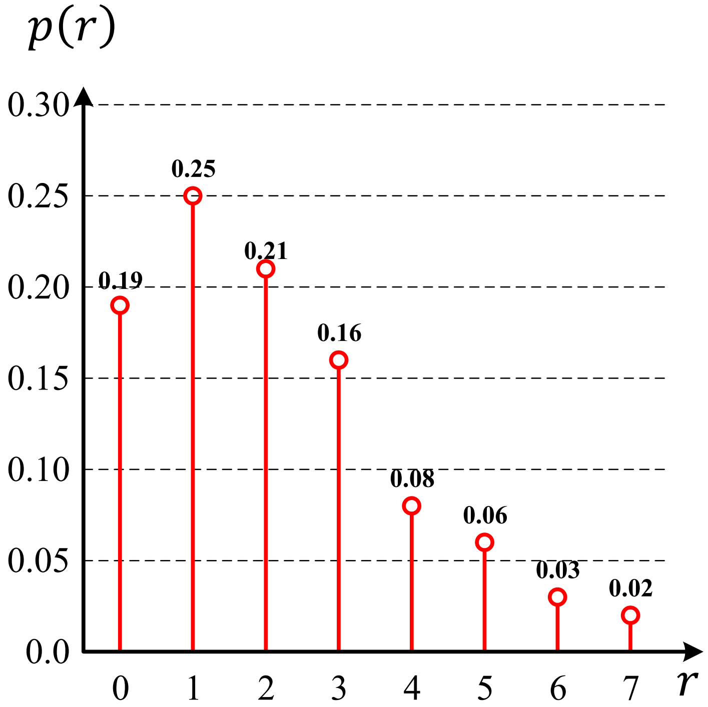
$$T(r=0) = 7 \times (0.19) = 1.33 \rightarrow 1$$

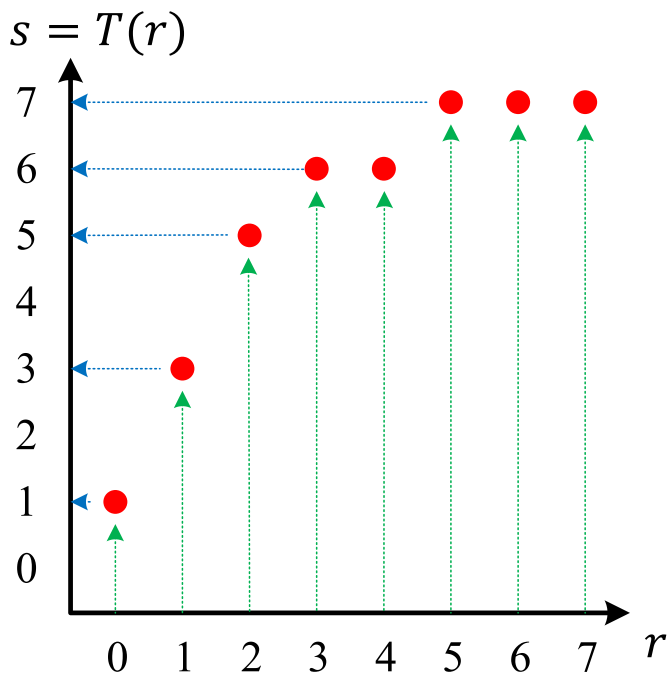
$$T(r=1) = 7 \times (0.19 + 0.25)$$
$$= 3.08 \rightarrow 3$$

$$T(r=7) = 7 \times \underbrace{(0.19 + \cdots + 0.02)}_{=1}$$
$$= 7$$

Histogram Equalization

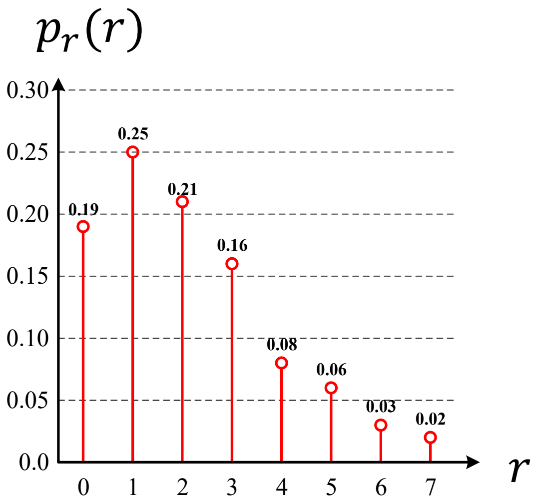
- Example: $T(.): p(r) \rightarrow$ uniform



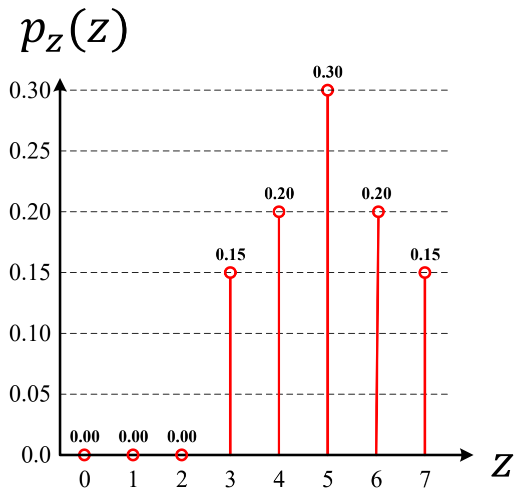


Matching Algorithm

- Problem Formulation
  - Given: source histogram $p_r(r)$ and target histogram $p_z(z)$
  - Goal: find a transformation from $p_r(r)$ to $p_z(z)$



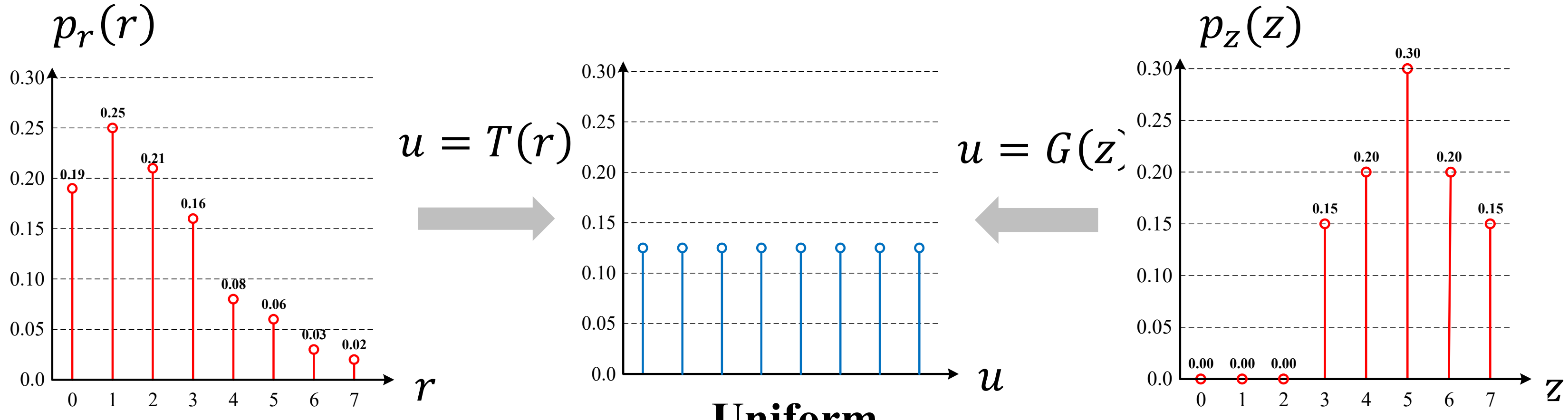
source histogram



target histogram

Matching Algorithm

- Algorithm Steps
  - Step 1: apply equalization algorithm to find $T(.)$ that transforms $p_r(r)$ to uniform distribution
  - Step 2: apply equalization algorithm to find $G(.)$ that transforms $p_z(z)$ to uniform distribution



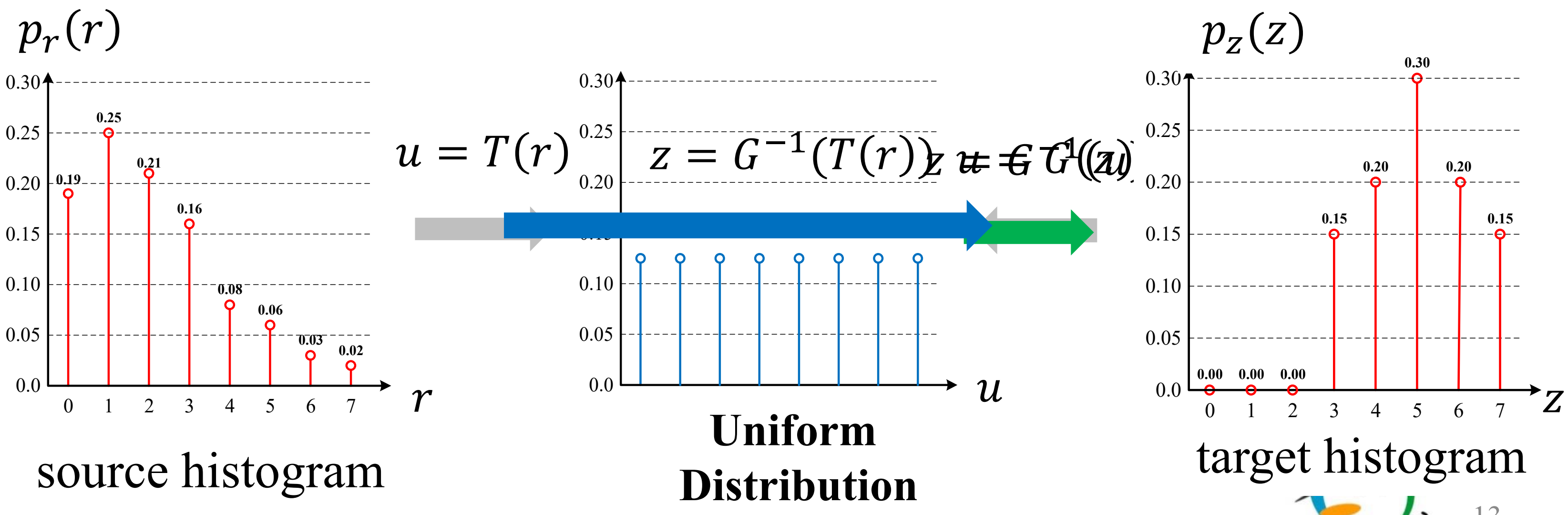
source histogram

Uniform Distribution

target histogram

Matching Algorithm

- Algorithm Overview
  - Step 3: compute inverse function $z = G^{-1}(u)$
  - Step 4: form the function $z = G^{-1}(T(r))$ and uses it for intensity transformation

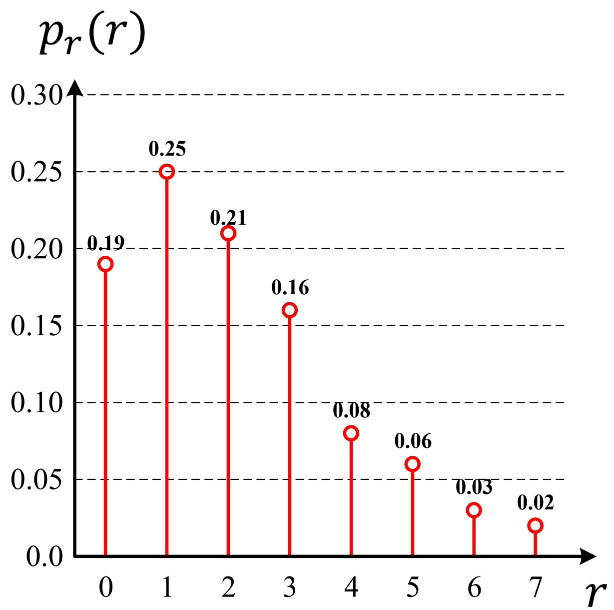


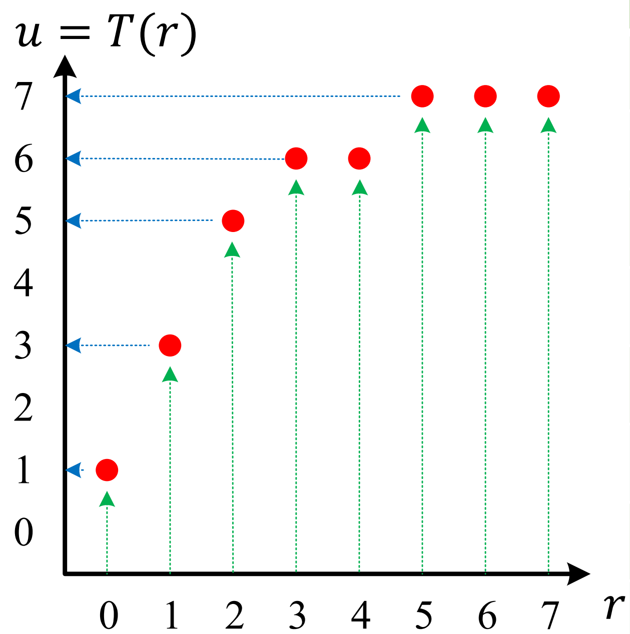
source histogram

target histogram

Matching Algorithm

- Step 1: find $T(.)$: $p_r(r) \rightarrow u$ (uniform)



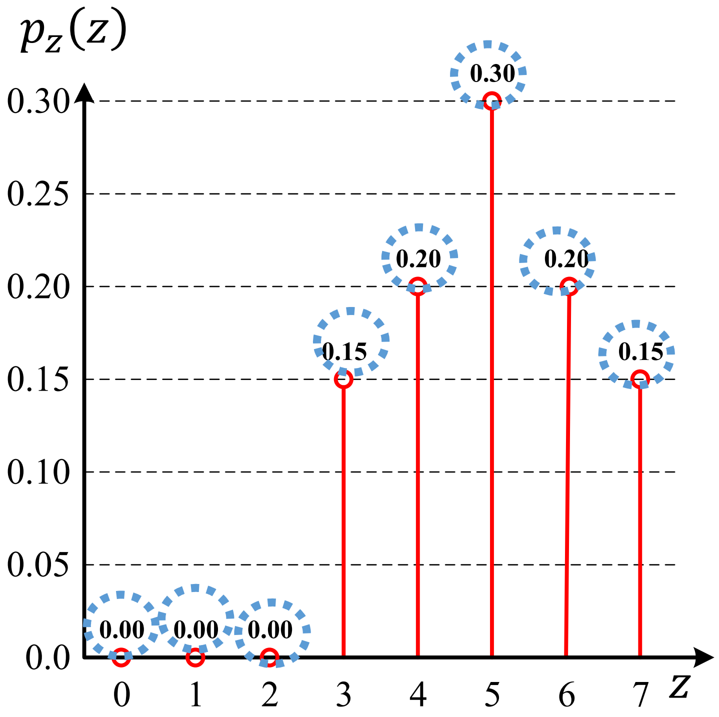


r	$u = T(r)$
0	1
1	3
2	5
3	6
4	6
5	7
6	7
7	7

Matching Algorithm

- Step 2: find $G(.)$: $p_z(z) \rightarrow u$ (uniform)

$$G(z=k) = (L-1) \times \sum_{j=0}^{k} p_z(z=j)$$



$$G(z=0) = 7 \times (0.00) = 0$$

$$G(z=1) = 7 \times (0.00 + 0.00) = 0$$

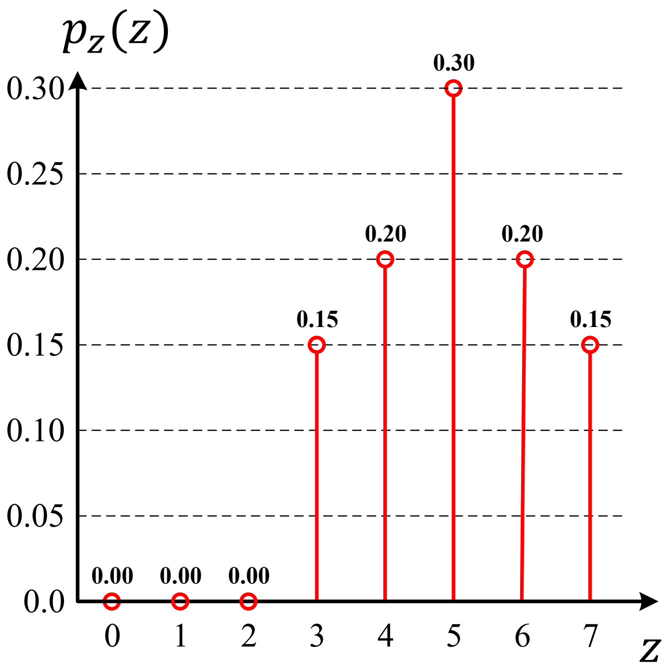
⋮

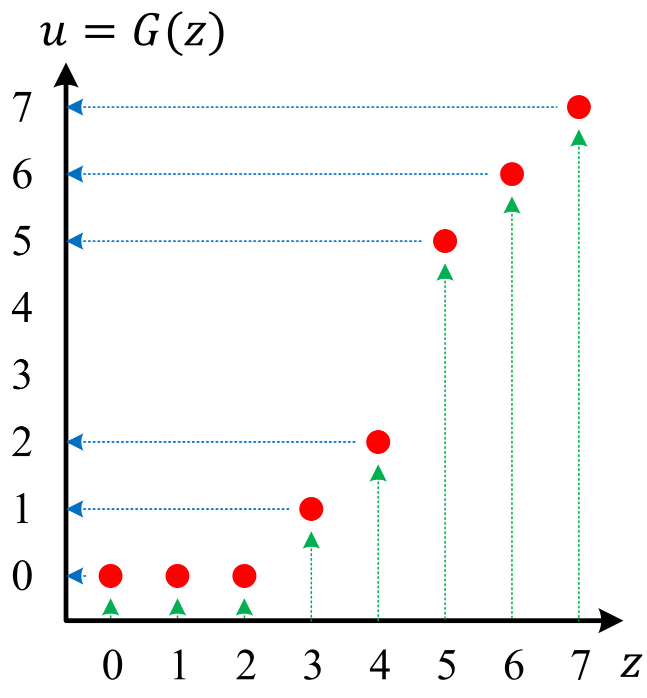
$$G(z=7) = 7 \times (0.00 + \cdots + 0.15)$$

$$= 7$$

Matching Algorithm

- Step 2: find $G(.)$: $p_z(z) \rightarrow u$ (uniform)

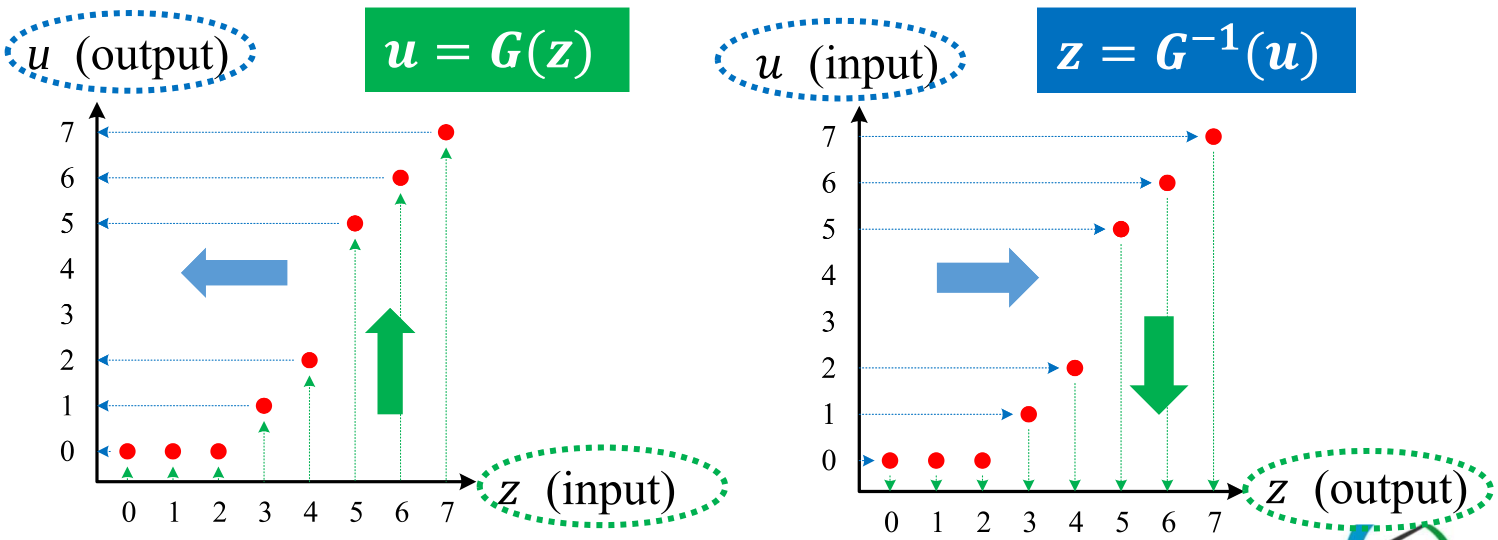




z	$u = G(z)$
0	0
1	0
2	0
3	1
4	2
5	5
6	6
7	7

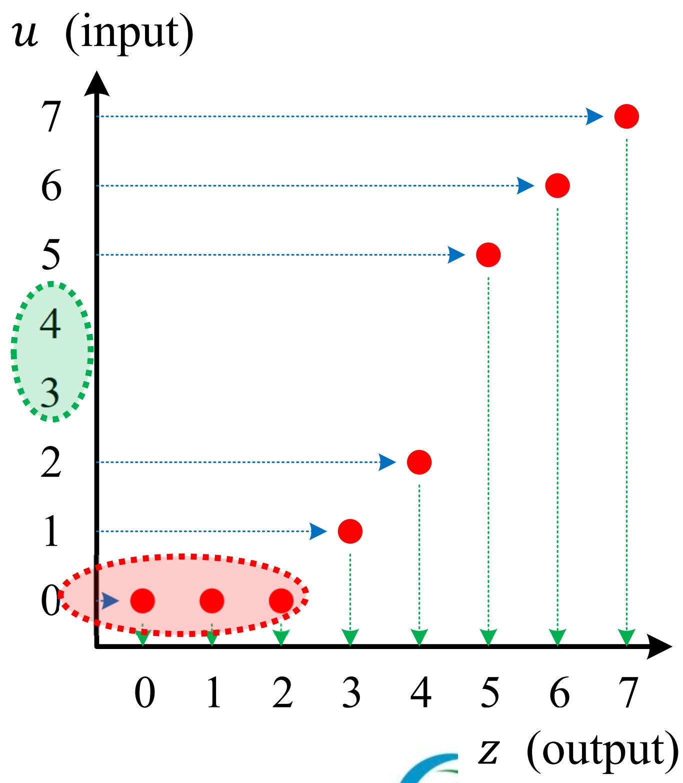
Matching Algorithm

- Step 3: compute $z = G^{-1}(u)$
  - compute $z = G^{-1}(u)$ by exchanging input and output of $u = G(z)$



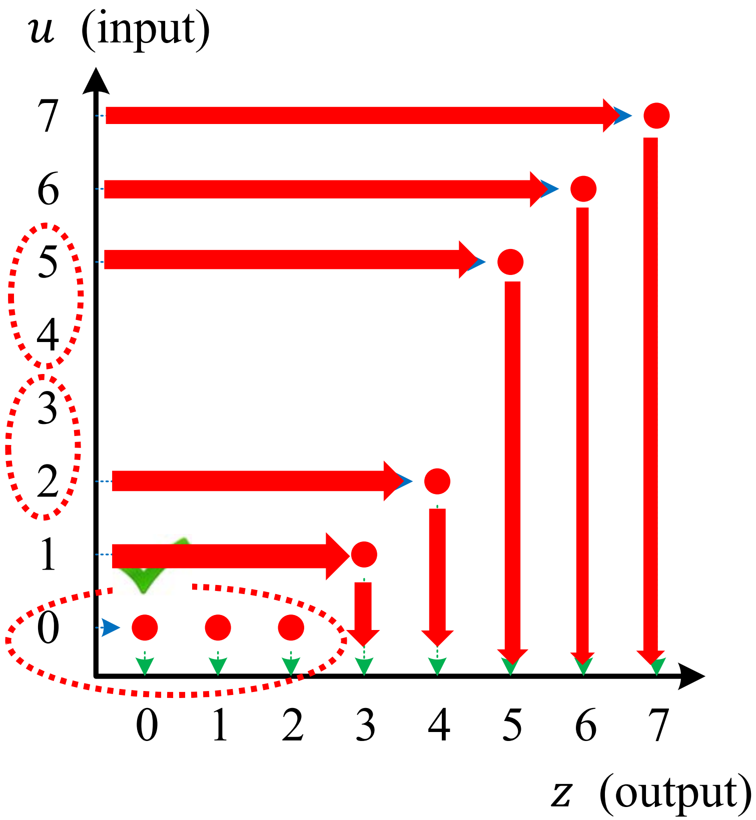
Matching Algorithm

- Step 3: compute $z = G^{-1}(u)$
  - Case 1: mapping is not unique: choose the smallest one z for output by convention
  - Case 2: no mapping exists: use the output of the u value that is the closet to current one.



Matching Algorithm

- Step 3: compute $z = G^{-1}(u)$



u	$z = G^{-1}(u)$
0	0
1	3
2	4
3	4
4	5
5	5
6	6
7	7

Matching Algorithm

- Step 4: form $z = G^{-1}(T(r))$ and do mapping

r	$u = T(r)$
0	1
1	3
2	5
3	6
4	6
5	7
6	7
7	7

u	$z = G^{-1}(u)$
0	0
1	3
2	4
3	4
4	5
5	5
6	6
7	7

r	$z = G^{-1}(T(r))$
0	3
1	4
2	5
3	6
4	6
5	7
6	7
7	7

Matching Algorithm

- Step 4: use $G^{-1}(T(.))$ for intensity mapping

0	1	2	3
0	3	1	1
6	2	7	5
1	4	5	1

input image

r	$z = G^{-1}(T(r))$
0	3
1	4
2	5
3	6
4	6
5	7
6	7
7	7

3	4	5	6
3	6	4	4
7	5	7	7
4	6	7	4

output image


thank you
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